Numerical investigation of shot noise suppression in chaotic cavities

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Abstract

We perform a numerical investigation of the suppression of shot noise, with respect to the full shot value predicted by Schottky's theorem, in chaotic cavities with ballistic or diffusive transport, as well as in the presence of a magnetic field. Results are discussed with reference to the recent theoretical and experimental literature.

Introduction

In the last decade, the field of shot noise in mesoscopic devices has received significant attention, due to a series of extremely interesting theoretical and experimental results. Jalabert et al. [1] showed, using random matrix theory, that shot noise in a chaotic cavity defined by two constrictions is suppressed, with respect to the value predicted by Schottky's theorem [2], by a factor (Fano factor) depending on the ratio of the number of modes propagating in the input constriction to that in the output constriction: if the two constrictions are identical, the suppression factor is 1/4. Blanter and Sukhorukov [3] have later presented a semi-classical demonstration of the same result, using a Boltzmann-Langevin approach. These results have received experimental confirmation with an experiment by Oberholzer et al. [4], in which shot noise suppression by a factor 1/4 has been measured for a symmetric cavity defined by means of electrostatic depletion in a GaAs/AlGaAs modulation doping heterostructure. More recently, Sukhorukov et al. [5] have investigated the Fano factor for cascaded cavities in the assumption of phase incoherence between different cavities. In this contribution, instead, we present a discussion of the behavior of shot noise for a fully phase coherent structure containing one chaotic cavity or two cascaded cavities. We consider also the case of shot noise in a cavity in which transport is diffusive as a consequence of a chaotic cavity or two cascaded cavities.

We have used two different numerical models for simulations with and without magnetic field. In the absence of magnetic field we use an approach based on the calculation of the Green's functions of the complete structure by means of the recursive composition [7,8] of elementary sections, for each of which the Green's functions can be evaluated analytically. Once the Green's function matrix for propagation from a location on the input lead to a location on the output lead is available, it is straightforward [7] to compute the transmission and reflection matrices, which contain the information needed to evaluate the Fano factor, as discussed later in this section. The recursive Green's function method is particularly convenient for the simulation of structures combining hard walls and a complex geometry, since it does not involve matching wave functions and their normal derivatives, which requires particular care at hard-wall boundaries [9].

For the simulations in the presence of a magnetic field, we have instead adopted the scattering matrix technique. In particular, to analyze a 2-dimensional device on the x,y plane (assuming x as the direction of current propagation) and subject to a uniform orthogonal magnetic field , we have used the specific gauge for the vector potential suggested by Governale and Böse [10]: . We have defined a uniform discretization along the x direction, with a resolution such that the vector potential and the scalar potential have negligible variations along x within a discretization mesh. Therefore, within each transverse section of width , the vector potential is constant, and the scalar potential depends only on y. With our choice of gauge, by solving the Schrödinger equation in each transverse section located around x, we find that the n-th eigenmode has an energy equal to

\[ E_{T \alpha \xi} \]

(the eigenenergy in the absence of a magnetic field) and a transverse eigenfunction

\[ \chi_{n,\xi}(y) \]

which differs only for a phase factor from the eigenfunction

\[ \chi_{0,\xi}(y) \]

for ; in particular

\[ \chi_{\alpha,\xi}(y) = \chi_{0,\xi}(y) e^{i A_{\xi}/\hbar} \]

where is the elementary electron charge, is Planck's reduced constant and

\[ A_{\xi} \]

is the value of the vector potential in the -th transverse section. For the application of the scattering matrix method, we have divided the structure into slices, each of which extends from to , thereby straddling two consecutive transverse sections. Therefore each slice contains only one discontinuity, located exactly in the
middle. Using the mode-matching technique [9], we have obtained the reflection and transmission matrices for the electrons impinging onto the left side of the slice ($r$ and $t$) and for those impinging onto the right side ($\bar{r}$ and $\bar{t}$).

Therefore we can construct the scattering matrix $S$ of the slice

$$S = \begin{bmatrix} r & \bar{r} \\ \bar{t} & t \end{bmatrix}$$

In order to limit discretization errors, we need to choose $\Delta x$ in such a way that the magnetic field flux threading a slice is much less than $h/2\pi e$, so that discretization errors are made negligible. Then we have recursively combined the scattering matrices of all the slices, obtaining the scattering matrix of the overall structure. From its sub-matrix $t$, we have derived the matrix $t'$ of the transmission coefficients normalized with respect to the probability current flux, the generic element of which is

$$t'_{nm} = t_{mn} \sqrt{v_n/v_m}$$

(with $v_n$ being the group velocity of the $n$-th mode). Finally, we have computed the value of the conductance

$$G = \frac{2e^2}{h} \sum_{n,m} |t'_{nm}|^2$$

and of the power spectral density of the shot noise

$$S_f = \frac{4}{\pi} e^2 V \sum_n w_n (1 - w_n)$$

(with $w_n$ being the eigenvalues of the matrix $tt'$ and $V$ the externally applied voltage), following Büttiker’s approach [11]. Knowing that the full shot noise power spectral density is given by $2e^2/2\pi e V|G$, the Fano factor $\gamma$ can be computed as

$$\gamma = \frac{\sum_n w_n (1 - w_n)}{\sum_n |t'_{nm}|^2}$$

**Results**

We have initially tested our approach retrieving the known result [1] for shot noise suppression in a single chaotic cavity with a width of 5 $\mu$m and a length of 7.5 $\mu$m; the input and output constrictions are identical and 1 $\mu$m wide, 0.25 $\mu$m long. The Fano factor is reported in fig. 1 as a function of the Fermi energy of the impinging electrons, while the inset contains a sketch of the cavity geometry.

As the Fermi energy is increased, the number of propagating modes increases, so that a chaotic behavior ensues and the Fano factor fluctuates around the value 1/4. In our calculations we consider a maximum of about 60 propagating modes, plus a few tens of evanescent modes. The fluctuations are due to the complex interference patterns that are established in the chaotic cavity: they would be damped in a model including dephasing effects.

Then we have investigated the shot noise suppression in two cascaded cavities, whose geometry is sketched in the inset of fig. 2: the dimensions of the cavities and of the constrictions are the same as those for the single cavity. Results are reported in fig. 2: we notice that the same suppression factor 1/4 is obtained as for the single cavity. This result differs from that in Ref. [5], which is obtained in the hypothesis of phase incoherence between the cavities. Further work is needed to determine what happens if only partial phase coherence exists and what the actual situation in an experimental device is.

Figures 1-3 - Fano factor for two coherent cascaded cavities as a function of the Fermi energy; the inset contains a sketch of the device. The dashed line indicates the theoretical value 1/4.

We have then investigated the shot noise suppression in a cavity with diffusive transport, caused by the presence of 400 randomly located scatterers, each of which consists of a square hard-wall obstacle with a side of 50 nm. The resulting potential landscape is sketched in fig. 3.

The cavity is 5 $\mu$m wide, 7.75 $\mu$m long, and the constrictions are 1 $\mu$m wide and 0.75 $\mu$m long. The Fano factor is reported as a function of the Fermi energy of the impinging electrons in fig. 4: we notice that the suppression factor is almost 1/3, as in a purely diffusive wire [12]. Thus, diffusive scattering prevails and the chaotic behavior of the cavity is quenched.

For the investigation of the noise behavior of a cavity in the presence of a magnetic field, we have applied the scattering matrix technique outlined in the previous section to a cavity with the same geometrical dimensions as the one studied for the diffusive case, but without the randomly located scatterers, and with the inclusion of a uniform magnetic field perpendicular to the plane containing the device.

We report, in fig. 5, the behavior of the conductance through the cavity as a function of the Fermi energy, for different values of the magnetic field: $B=0$, 7, 14, 21 mT. We observe that for the case with $B=0$ the conductance shows fast oscillations that are due to interference effects, and that the conductance steps are completely washed out. In this calculation we have considered up to about 35 propagating modes and a few tens of evanescent modes.
If we compare the average conductance for a given value of the Fermi energy with that computed, in the same condition, for a single constriction, we find that it corresponds to one half of such a value: in other words the resulting resistance equals the series of the resistances due to the constrictions, as expected [1]. As the magnetic field is increased, we notice that conductance quantization is progressively recovered and that conductance steps have an amplitude corresponding to one single conductance quantum. This can be interpreted in terms of a semiclassical model similar to that of Ref. [6]: in the presence of a magnetic field, electrons tend to travel in cyclotron orbits, whose radius $R$ is given by $R = \frac{mv}{eB}$, where $v$ is the electron velocity and $m$ is the effective mass. As the cyclotron radius becomes comparable with the size of the cavity, skipping orbits start forming along its walls and the chaotic behavior of the cavity begins to disappear. For values of the magnetic field that yield orbits small compared to the size of the cavity, transport is mediated by edge states crawling along the perimeter and the overall conductance corresponds to that of a single constriction. Conductance quantization is first recovered for lower values of the energy, since they correspond to a smaller cyclotron radius, if we consider the electron velocity for the calculation of the cyclotron radius equal to the Fermi velocity. For the cavity size being taken into account and the considered range of energies, we find that edge states are formed for values of the magnetic field of the order of tens of millitesla. This value differs from that reported in Ref. [6]: the reasons for this discrepancy should be investigated more in detail, since they are in part to be attributed to the limited value of the Fermi energy that we have considered in order to contain the size of the numerical problem, and also to details of the experiment, such as the effective geometry of the cavity on which measurements have been performed. We also notice, for the largest magnetic field reported in fig. 5, that the energy intervals spanned by each conductance step tend to be of equal amplitude, instead of exhibiting the quadratic increase expected from hard-wall confinement. This is the consequence of the effective parabolic confinement due to the action of the magnetic field [13], which leads to a uniform spacing of the eigenvalues. In fig. 6 we show the Fano factor as a function of the Fermi energy for the same series of values of the magnetic field.

The suppression factor, oscillating around $1/4$ for $B=0$, declines as $B$ is increased, until shot noise is completely quenched for the energy intervals in which full conductance quantization is recovered.

The reason for noise disappearance is the same as that for the reappearance of the conductance steps, i.e. the formation of edge states that crawl along the cavity walls without giving rise to chaotic behavior.

**Conclusions**

We have presented a numerical investigation of shot noise suppression in chaotic mesoscopic cavities, including the effect of diffusive regions and of a perpendicular magnetic field. The recursive Green's function method has been used for the investigation of the structures without magnetic field, while in the presence of a magnetic field calculations have been performed with the scattering matrix approach and the gauge choice of Ref. [10]. We find that, in the hypothesis of complete phase coherence, two cascaded cavities exhibit the same shot noise suppression factor of $1/4$ as a single cavity and that, if randomly placed scatterers are added in a cavity to make transport diffusive, the Fano factor is very close to that for a diffusive wire, without any constriction. We then focus on the analysis of the conductance and noise behavior in the presence of a magnetic field, observing that, as the magnetic field increases and edge states start forming, conductance quantization is gradually recovered and noise is quenched, due to the disappearance of the chaotic behavior. Further work is planned to compare with experimental results obtained on specific devices, and to introduce partial decoherence effects.

**References**